Content Complexity for Mathematics and Science Instructional Planning

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FLORIDA STATE UNIVERSITY
2012 CONFERENCE
HILTON ST. PETERSBURG BAYFRONT, DECEMBER 6 – 8, 2012

Outline of Presentation

Part 1	**Content Complexity and Depth-of-Knowledge**
Part 2	**Common Core State Standards and Assessment Tasks**
Part 3	**Instructional Strategies**

Content Complexity

Differentiates learning expectations and outcomes by considering the amount of prior knowledge, processing of concepts and skills, sophistication, number of parts, and application of content structure required to meet an expectation or to attain an outcome.

Bloom Taxonomy

Knowledge	Recall of specifics and generalizations; of methods and processes; and of pattern, structure, or setting.
Comprehension	Knows what is being communicated and can use the material or idea without necessarily relating it.
Applications	Use of abstractions in particular and concrete situations.
Analysis	Make clear the relative hierarchy of ideas in a body of material or to make explicit the relations among the ideas or both.
Synthesis	Assemble parts into a whole.
Evaluation	Judgments about the value of material and methods used for particular purposes.

The Cognitive Processing Dimension of the Revised Bloom's Taxonomy

- ✓ Remember
- ✓ Understand
- ✓ Apply
- ✓ Analyze
- ✓ Evaluate
- ✓ Create

The Separate Knowledge Dimension of the Revised Bloom's Taxonomy

Factual Knowledge - The basic elements students must know to be acquainted with a discipline or solve problems in it.

Conceptual Knowledge - The interrelationships among the basic elements within a larger structure that enable them to function together.

Procedural Knowledge - How to do something, methods of inquiry, and criteria for using skills, algorithms, techniques, and methods.

Metacognitive Knowledge - Knowledge of cognition in general as well as awareness and knowledge of one's own cognition.

Depth of Knowledge (1997)

Level 1 Recall

Recall of a fact, information, or procedure.

Level 2 Skill/Concept

Use information or conceptual knowledge, two or more steps, etc.

Level 3 Strategic Thinking

Requires reasoning, developing plan or a sequence of steps, some complexity, more than one possible answer.

Level 4 Extended Thinking

Requires an investigation, time to think and process multiple conditions of the problem.

Subject	Depth of Knowledge			
	Level 1	Level 2	Level 3	Level 4
Mathematics	Examples: ◆ **Count to 100 by ones and by tens.** ◆ **Fluently multiply and divide within 100**, using strategies such as the relationship between multiplication and division ◆ **Write simple expressions** that record calculations with numbers, and interpret numerical expressions without evaluating them (e.g. Recognize that 3 × (18932 + 921) is three times as large as 18932 + 921).	Examples: ◆ **Measure and estimate liquid volumes** and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). ◆ **Apply properties of operations** as strategies to add and subtract rational numbers. ◆ **Measure and record data** and **produce graphs** of relevant variables.	Examples: ◆ **Analyze similarities and differences between procedures** ◆ **Formulate original problem** given situation ◆ **Formulate mathematical model** for complex situation ◆ **Interpret the rate of change** and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values	Examples: ◆ **Derive a mathematical model** over multiple days to explain a complex phenomenon or make a prediction. ◆ **Conduct a project** that specifies a problem, identifies solution paths, solves the problem, and reports results ◆ **Design a mathematical model** to inform and solve a practical or abstract situation.

Subject	Depth of Knowledge			
	Level 1	Level 2	Level 3	Level 4
Science	Examples: ◆ **Recall or recognize a fact, term, or property**. ◆ **Provide or recognize a standard scientific representation** for simple phenomena. ◆ **Perform a routine procedure** such as measuring length. ◆ **Identify familiar forces** (e.g. pushes, pulls, gravitation, friction, etc.) ◆ **Identify objects and materials as solids, liquids, or gases**.	Examples: ◆ **Specify and explain the relationship** among facts, terms, properties, and variables. ◆ **Identify variables, including controls**, in simple experiments. ◆ **Distinguish between experiments and systematic observations**. ◆ **Describe and explain examples and non-examples of science concepts.**	Examples: ◆ **Identify research questions and design investigations f**or a scientific problem. ◆ **Design an experiment or systematic observation** to test a hypothesis or research question. ◆ **Develop a scientific model for a complex situation.** ◆ **Form conclusions from experimental data.** ◆ **Explain how political, social, and economic concerns can affect science, and vice versa.**	Examples: ◆ Based on provided data from a complex experiment that is novel to the student, **deduce the fundamental relationships among several variables.** ◆ **Conduct an investigation, from specifying a problem to designing and carrying out an experiment, to analyzing its data and forming conclusions, and write a report.**

$$\begin{array}{r} 121 \\ 13 \\ 32 \\ +\ 34 \\ \hline \end{array}$$

1) 190
2) 200
3) 290
4) N

Which of these means about the same as the word *gauge*?

a. balance

b. measure

c. select

d. warn

A car odometer registered 41,256.9 miles when a highway sign warned of a detour 1,200 feet ahead. What will the odometer read when the car reaches the detour? (5,280 feet = 1 mile)

(a) 42,456.9

(b) 41,279.9

(c) 41,261.3

(d) 41,259.2

(e) 41,257.1

Did you use the calculator on this question?

☐ Yes ☐ No

A triangle has 0 diagonals, a quadrilateral has 2 diagonals, a pentagon has 5 diagonals, and a hexagon has 9 diagonals. If the pattern continues, how many diagonals will a octagon have?

Sides	3	4	5	6
Diagonals	0	2	5	9

A. 11
B. 14
C. 18
D. 20

DOK 2

A scientist synthesizes a new drug. She wants to test its effectiveness in stopping the growth of cancerous tumors. She decides to conduct a series of experiments on laboratory mice to test her hypothesis.

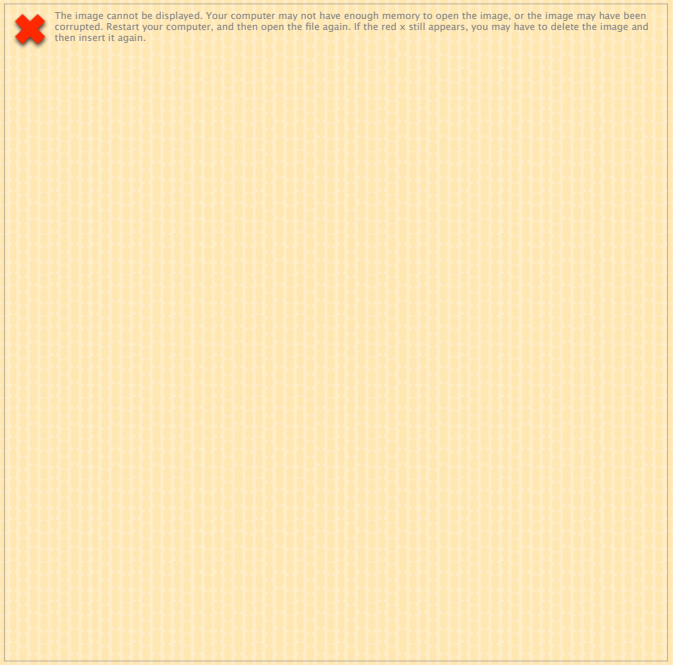
What should she do?

A. Give half the mice the drug, the other half none, and compare their tumor rates.
B. Give the drug to all mice, but only to half every other day, and record tumor rates.
C. Double the dosage to all mice each day until tumors start to disappear.
D. Give the drug only to those mice that have tumors and record their weights.

DOK 2

Grade 8 Mathematics Task

Look at the drawing. The numbers alongside each column and row are the total of the values of the symbols within each column and row. What should replace the question mark?



DOK 3

It Is Still A Level 1

Marc Umile poses for a picture in front of a projection of the string of numbers knows as pi in Philadelphia, Friday, March, 2, 2006. Umile is among a group of people fascinated with pi, a number that has been computed to more than a trillion decimal places. He has recited pi to 12,887 digits, perhaps the U.S. record. (AP Photo/Matt Rourke)

Content Complexity is Continuous and Generally Decided by Content Analyses.

COMMON CORE STANDARDS

Mathematics

Standards for Mathematical Practice

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Structure of CCSS Mathematics

Critical Areas

Domains

Clusters

Standards

GR 3 Domain, Cluster, Standards

Number and Operations in Base Ten 3.NBT

Use place value understanding and properties of operations to perform multi-digit arithmetic.

1. Use place value understanding to round whole numbers to the nearest 10 or 100.
2. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.
3. Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 × 80, 5 × 60) using strategies based on place value and properties of operations.

Grade 5 Number and Operations-Fractions

Use equivalent fractions as a strategy to add and subtract fractions.

1. Add and subtract fractions with unlike denominators (including mixed numbers) denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like, a/b + c/d = (ad + bc)/bd.)

2. Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7 by observing that 3/7 < 1/2.

Depth of Knowledge of All Mathematics Common Core State Standards

Grade	Total	DOK Level			
		1	2	3	4
3	25	24	24	3	0
4	28	28	16	5	0
5	26	26	18	3	0
6	29	29	20	2	0
7	24	18	22	8	0
8	28	26	25	9	0
Number and Quantity	27	27	15	0	0
Algebra	27	26	21	7	0
Functions	28	27	24	4	0
Geometry	43	24	36	19	1
Statistics and Probability	31	27	29	7	0
TOTAL	316	282	250	67	1
Percentage of Total Standards at DOK Level (Standards may cover a range of DOK levels)		**89%**	**79%**	**21%**	**< 1%**

Sato, E, Lagunoff, R, and Worth, P. (March 4, 2011) SMARTER Balanced Assessment Consortium Common Core State Standards analysis: Eligible content for the summative assessment. Final report submitted to the SMARTER Balanced Assessment Consortium. San Francisco, CA: West Ed.
http://www.k12.wa.us/SMARTER/pubdocs/SBAC_CCSS_Eligible_Content_Final_Report_030411.pdf downloaded October 17, 2011.

FLORIDA ANALYSIS OF DOK LEVELS OF CCSS MATHEMATICS FOR HIGH SCHOOL

	Total # Standards	DOK Level			
		1	2	3	4
Number and Quantity	27	13	14		
Functions	28	2	23	3	
Algebra	27	7	14	6	
Geometry	43	5	24	14	
Statistics and Probability	31	3	27	1	
Total	156	30	102	24	
Percent		19%	65%	15%	
Smarter Balance %		84%	80%	24%	1%

PARCC ASSESSMENT TASK TYPES

TYPE I: Tasks Assessing Concepts, Skills, and Procedures

- **Conceptual understanding, fluency, and application**
- **Involve any or all mathematical practice standards**
- **Machine Scorable**

TYPE II: Tasks Assessing Expressing Mathematical Reasoning

- **Written arguments/justifications, critique of reasoning, or precision in mathematical statements**
- **Mix of innovative, machine scored and hand scored responses.**

TYPE III: Tasks Assessing Modeling Applications

- **Modeling/application in a real-world context or scenario**
- **Mix of innovative, machine scored and hand scored responses**

SMARTER BALANCE SAMPLE ITEM FOR CCSS: 4.MD.3

43022

A rectangle is 6 feet long and has perimeter of 20 feet. What is the width of the rectangle? Explain how you solved this problem.

SMARTER BALANCE SAMPLE ITEM FOR CCSS: 4.MD.3

43023

A rectangle is 6 feet long and has perimeter of 20 1/3 feet. What is the width of the rectangle? Explain how you solved this problem.

RUBRIC FOR SCORING ITEM 43022-TWO PTS

Rectangle 1
Sample Top-Score Response:
20 – 6 – 6 = 8 feet
Half of 8 feet is 4 feet, so the width is 4 feet long.

Full credit (2 points):
The response demonstrates a full and complete understanding of problem solving.

The response contains the following evidence:
- **The student determines that 4 feet is the width of the rectangle with a correct process clearly demonstrated.**

RUBRIC FOR SCORING ITEM 43022-ONE PT

Partial credit (1 point):
The response demonstrates a partial understanding of problem solving. The response contains the following evidence:

- **The student determines 4 feet is the width, but does not show sufficient work to support this conclusion.**

OR

- **The student begins a correct process for determining the missing width, but ends up with an incorrect solution due to an incomplete process, computational mistake, or other mechanical error in the process.**

TARGETED STANDARD FOR ITEMS 43022 AND 43023

CCSS 4.MD.3

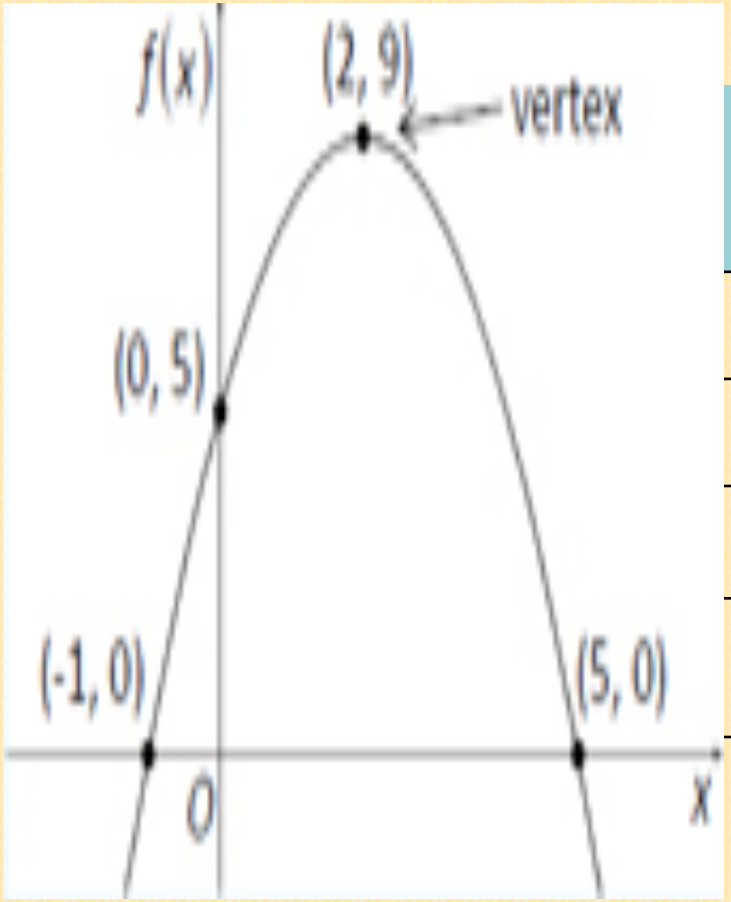
Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.

FL DOK 2

SB DOK 1 and 2

PARCC SAMPLE MATHEMATICS ITEM HIGH SCHOOL FUNTIONS

A portion of the graph of a quadratic function $f(x)$ is shown in the xy - plane. Selected values of a linear function $g(x)$ are shown in the table.



x	g(x)
-4	7
-1	1
2	-5
5	-11

For each comparison below, use the drop-down menu to select a symbol that correctly indicates the relationship between the first and the second quantity.

First Quantity	Comparison	Second Quantity
The y-coordinate of the y-intercept $f(x)$		The y-coordinate of the y-intercept $g(x)$
$f(3)$		$g(3)$
Maximum value of $f(x)$ on the interval $-5 \le x \le 5$		Maximum value of $g(x)$ on the interval $-5 \le x \le 5$
$(f(5) - f(2))/(5 - 2)$		$(g(5) - g(2))/(5 - 2)$

TARGETED STANDARD FOR PARCC SAMPLE MATHEMATICS ITEM

CCSS F-IF.9

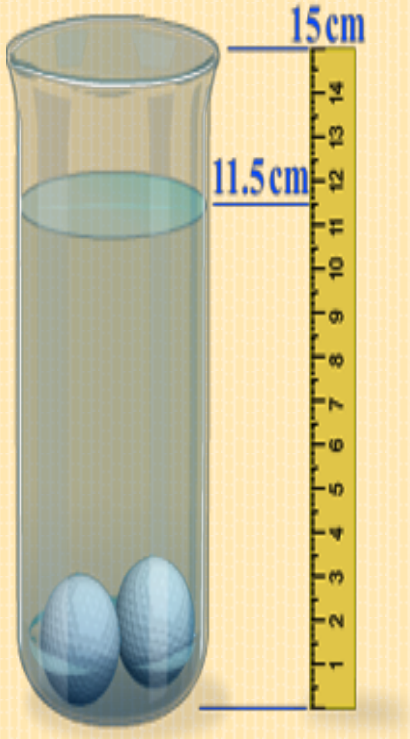
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

FL DOK 2

SB DOK 1 and 2

Golf Balls in Water—Dana Center

Tom is doing an experiment adding golf balls to a glass jar containing water. The picture and the table show what happens to the height of the water as Tom adds golf balls.



Number of golf balls, x	Height of water in centimeters, y
0	9.0
1	10.2
2	11.5
3	12.7
4	13.8

Drag tiles to complete the sentences and the equation below based on the results of Tom's experiment.

golf balls	change	glass jars	water height	1.16
1.2	1.3	9.0	12.0	13.8

The height of the water changes at an average rate about [ ] centimeters per golf ball. If these data are graphed with the number of golf balls as the independent variable, the **y**-intercept for the graph would be about [ ] centimeters. This means that for zero [ ], the [ ] is 9 centimeters. Tom's table and graph can be represented by the trend line with the equation

y = [ ] x + [ ]

TARGETED STANDARD FOR PARCC SAMPLE MATHEMATICS ITEM

Linear, Quadratic, and Exponential Models★ F-LE

Construct and compare linear, quadratic, and exponential models and solve problems.

2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

5. Interpret the parameters in a linear or exponential function in terms of a context.★

FL DOK 2 SB DOK 1, 2, and 3

Strategies For Using DOK To Improve Learning

Deconstruction of a Standard

2.OA.3. Determine whether a group of objects (up to 20) has an odd or even number of members, e.g., by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends. (Grade 2 Operations and Algebraic Thinking)

DOK 1:

Define:

- odd
- even
- number
- addend
- group
- equation
- equal
- sum

Count to 100

Count by 2

Deconstruction (continued)

DOK 1 (continued):

Understand the number line

Place numbers on a number line

Use skip counting

Pair objects

Compare numbers

Identify a number as odd or even

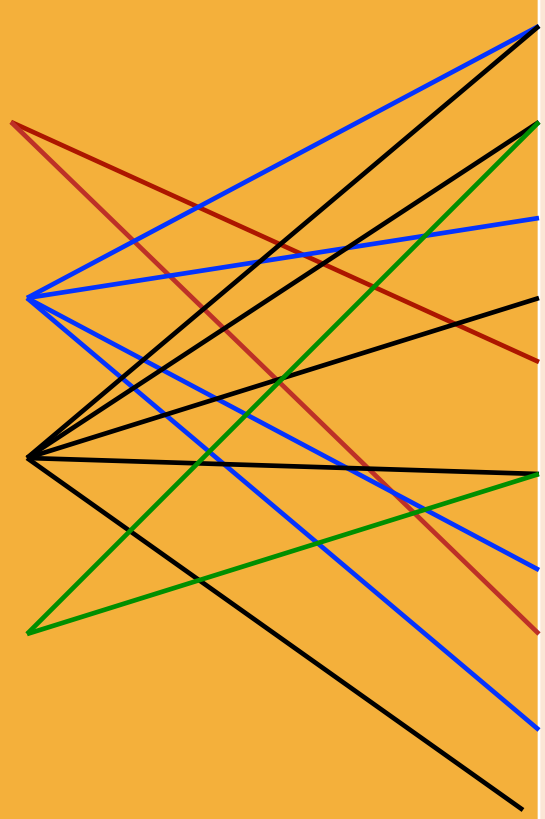
Identify an equation

DOK 2:

Write an equation

Match Instructional Activity To Complexity of Outcome

Complexity of Outcome	Instructional Activity for Student
DOK 1 DOK 2 DOK 3 DOK 4	Model Plan Paraphrase Justify Practice Search Build Repeat Represent Reveal thinking



Design Appropriate Assessments

- Question
  - DOK 1 What, Where, Find, Compute
  - DOK 2 Why or Why Not, Compare
  - DOK 3 Imply, Infer, Project, Generalize
- Context
  - DOK 1 None
  - DOK 2 Familiar and Relevant
  - DOK 3 Unfamiliar, Multiple, Entire Text
- Application
  - DOK 1-2 Little or none
  - DOK 3-4 Fit, verify, and justify

Issues with DOK

Issues in Assigning Depth-of-Knowledge Levels

- Complexity vs. difficulty
- Distribution by DOK Level
- Item type (MC, CR, OE, EBSR)
- Central performance in objective
- Consensus process in training
- Application to instruction
- Reliability

Web Sites

http://facstaff.wcer.wisc.edu/normw/

PARCC Web Site

http://www.parcconline.org/parcc-assessment